On Vortex Trails over Ocean Islands

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Abstract

In this study some experimental results are given to demonstrate that the density stratification is the main reason for the appearance of cloud vortex trails over ocean islands. Evidences are given that the similarity of these vortex trails to von Kàrmàn streets may not go beyond the general streak-line pattern. The experimental results reveal the three-dimensional vortex shedding structure when a sphere is towed at a constant velocity through a stratified fluid. It is found that for small or moderate stratification and Reynolds numbers in the range from 10^3 to 10^4 the vortex is shed three-dimensionally. The stratification however quickly and effectively inhibits the vertical motion and the initially turbulent wake collapses and reveals the vertically oriented portion of the vortex structure, reminiscent of the two-dimensional vortex street behind a circular cylinder when viewed from above. Considerable insights are given concerning the vortex shedding and its structure in the wake of a three-dimensional body.

1. introduction

In the early sixties a new and interesting flow phenomenon was discovered in the earth's atmosphere by meteorological satellites: the mesoscale vortex trails behind the islands of Madeira, Jan Mayen and Guadalupe (Baja California), to name only the three most significant locations. Satellite pictures of this kind show a range of mesoscale eddies (diameters of order 100 km) which appear to have been mechanically produced by mountains, i. e. hydrodynamically induced. Figure 1 is a satellite picture showing the von Kàrmàn street-like vortex trail caused by steady wind downstream of the island of Madeira. Figure 2 shows a chain of vortices from the lee side of the Canary Islands (Bugaev 1973). Figure 3 shows a similar vortex trail in the downstream of Guadalupe. In a first paper of Hubert and Krueger (1962), the vortex trails, revealed by the satellite pictures, were attributed to instabilities of the geostrophic wind stream, this instability being introduced by the vertical component of the flow in the neighborhood of an island serving as an obstacle. A later paper of Chopra and Hubert (1965) insinuates from the beginning that a similarity between the cloud vortex streets behind Madeira and the von Kàrmàn vortex streets produced from infinitely long

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cylinders exists. However, the island of Madeira at the level of the clouds has a width of about 30 km normal to the wind direction and its geometry is that of a flat hump, which could not by the most tolerant approach be regarded as a cylinder of appreciable aspect ratio. Although, by using the results of Birkhoff and Zarantonello's (1957) analysis of the diffusion and decay of a vortex in a two-dimensional vortex streets. Chopra and Hubert have established good agreement for the spacing ratio between laboratory vortex streets and those over Madeira. The similarity, nevertheless, fails in the case of an estimate of the circumferential velocity in the atmospheric vortices. They find for a vortex south of Madeira a circumferential velocity of 88 Knots at a radius of 20 km; such a speed in a big vortex could not possibly exist without being detected by aircraft! Only recently Moll (1971) has pointed out that the flat hump of tlhe isand of Madeira bears no similarity to a long circular cylinder.

On the other hand, the cloud formations as shown by those satellite pictures look very much the same as streak-line photographs of a two-dimensional wake behind a long circular cylinder (see, for example, Batchelor 1967). Moll investigates two kinds of inhomogeneity in the lower atmosphere that could be the origin of a two-dimensionality in the flow around Madeira. The first hypothesis is that we deal with a phenomenon in a rotating fluid in which a Taylor column is fixed to Madeira or part of it. After considering a range of Rossby number and Ekman number from the weather data taken at Funchal, Madeira, he concludes that the existence of a Taylor column over Madeira is improbable.

Moll's second hypothesis is that the two-dimensionality of the flow has its origin in a stable stratification of the atmosphere. He has given proof from meteorological data at the time of the satellite photographs that a stable stratification beneath the cloud layer did indeed exist. Moll points out that atmospheric vortex trails are phenomena occurring in a stratified fluid. These and other periodic flow phenomena were recently summarized in a review article by Berger and Wille (1972).

Very recently Pao and Kao (1975), and Kao and Pao (1975) have made a series of experiments, investigating the wake of a three-dimensional obstacle in a density stratified fluid. In this study some experimental results are given to demonstrate that the density stratification is indeed the sole reason for the appearance of these atmospheric vortex trails and to support the argument of Moll's that the similarity of these vortex trails to von Kàrmàn vortex streets may not go beyond the general streak-line pattern. Clarification is made as to why such a similarity exists. Considerable insights are also given concerning the vortex shedding and its structure in the wake of a three-dimensional body.

2. Experimental Observations

The experiments were performed in a long channel made of clear acrylic sheets. The dimensions of the channel were 14 inches wide, 24 inches deep, and 30 feet 6 inches long. Two kinds of techniques are used to study the wake of a three-dimensional obstacle in a density stratified fluid. The first method is the conventional method of towing the obstacle while the second method is new in which a uniform stratified flow is produced over a stationary obstacle in an open channel.

(A) Towing Experiment. The obstacle was attached to a towing carriage by means of a slender steel rod. The carriage was carefully designed to minimize mechanical vibrations when towed. The motion was initially accelerated gradually to avoid impulsive transients and the obstacle travelled several feet before the final steady towing speed was reached and maintained. The channel was filled with 18 one-inch layers of salt solutions of decreasing density by means of a floater. The procedure was the same as that described in Kao et al. (1974). The experiments were performed 20 hours after the filling process and the fluid was then linearly stratified. In the experiments, spheres were used as obstacles. For flow visualization the sphere was coated with nigrosine crystals arouad the forward stagnation point. When moving the dye marked out the wake region very effectively.

The pictures shown in Fig. 4 are from a representative experiment for a 2¼ in. dia. sphere. The sphere was towed at the mid-depth of the fluid at a steady speed U of 2.75 in/sec. This speed was maintained for 1 min and 30 sec. Pictures were taken from an overhead camera and a side-view camera simultaneously by means of a synchronous device. Both cameras were focused at a fixed location, so that the scenes were flow patterns at the same fixed location, looking from above and from the side, at various stages of development of the wake.

The stratification was characterized by the Brunt-Väisälä frequency N defined as

$$N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz},$$

where $\bar{\rho}(z)$ was the ambient density stratification and $\rho_0 = \bar{\rho}(0)$ and $z=0$ corresponds to the level of the sphere center. In the experiment, $N=0.567$ sec^{-1}, $U=2.75$ in/sec, $D=2.25$ in, so that the Reynolds number is $Re=UD/\nu=4{,}300$ and the Richardson number is $Ri=N^2D^2/U^2=0.215$.

Figure 4 shows the time development of the wake. In all the pictures the grid in the background is 1 in × 1 in and the shining dots are neutrally buoyant oil droplets used as velocity tracers.

In Fig. 4a we see the initial movement of the wake with the sphere in view. Simultaneous pictures of the top and side views are shown. The wake is seen to be fully turbulent and resembles that of the wake in a homogeneous fluid. This picture was taken 25 sec after the commencement of the expeiment.

Figure 4b are the top and side views at 119 sec after the sphere has passed the location. A beautiful double row vortex, very similar to von Kármán vortex street, is seen. The spacing between the vortices is about 14 inches when corrected for parallax. The side view now shows clearly the double sheeted structure of the dye.

(B) Flowing Experiment. A new method (Kao and Pao, 1975) was introduced to produce a uniform stratified flow over a stationary obstacle in an open channel. The flow is achieved by discharging the fluid from the channel through a sink. The details of the sink are unimportant. The flow speed is limited only by the sink capacity. Selective withdrawal at lower densimetric. Froude numbers is effectively eliminated through the use of a contraction. The standing free surface long wave due to flow initiation is also eliminated by the contraction. Experiments are conducted for flow over a sphere and an ellipsoid for a range of Reynolds numbers and Richardson numbers.

Figure 5 shows a top view of the wake of an ellipsoid which is just beyond the left edge of the picture. Again the double row vortex trail resembles von Kármán vortex street.

Figure 6 shows both top and side views of the wake behind a sphere. It is noted that a mirror, inclined at an angle of 45°, was mounted on top of the channel so that the lower part of the picture shows the side view while the upper part shows the top view which is actually the image in the mirror. It is seen that the thickness of the dye (about 1 in. here) is considerably thinner than the diameter of the sphere ($2\frac{1}{4}$ in.).

3. Double-Helical Vortex Loop and Its Collapse in a Stratified Fluid

It was proposed in an earlier paper by Pao and Kao (1975) that the vortex tube in the turbulent wake of a sphere in a homogeneous fluid has a close-ended double-helical structure. A picture of the helix model with circulations as indicated is shown in Fig. 7. A Schematic representation of the vortex configuration in the wake of a sphere is shown in Fig. 8. The term "double-helical" vortex loop is used somewhat loosely here and does not demand a regular double helix of constant pitch. It provides for the vortex tube to interact as the two branches cross each other. Such an interaction would lead to a nodal element where the strength of the circulation is double that of the individual branch of the vortex tube. It is important to emphasize that the double helix must form a close-ended loop in the fluid at the initiation of the process of vortex tube formation. The two branches of the double helix are then continuously releasing in an opposite sense from the cylindrical vortex sheet in the formation region as the process continues.

Now if the fluid is stratified, the stratification is known to inhibit the vertical fluctuations quite effectively (see, for example, Yih 1965). As a consequence, the initially turbulent wake collapses and spreads horizontally. As mentioned above the nodal elements of the vortex tube in the wake are elements around which the strength of the circulation is doubled. These elements of reinforced circulation were subsequently revealed by the stratification during the wake collapse process while the remaining portions of the helical system were dispersed. An examination of the model in Fig. 7 indicated that these points of enhanced circulation would be arranged in precisely the manner shown in Fig. 4b when viewed from above if the nodal elements were oriented approximately in a vertical direction. The double sheeted structure of the dye layers is perhaps due to the finite extent and slightly non-vertical orientation of the nodal elements. The stratification appears to play a role in restraining the orientation of the nodal elements to be nearly vertical but this remains to be confirmed through more careful experiments. What is indicated is that stratification of fluid effectively reveals the regularity of the originally three-dimensional turbulent wake which would otherwise appear to be turbulent and irregular if there is no stratification.

For comparison purpose flow visualization towing experiments were also conducted in a homogeneous fluid. The results for Re=4,300 is shown in Fig. 9. The dye structure again indicates the helical nature of the wake although the regular double-helical structure of the wake is largely overshadowed by the turbulent fluctuations there.

4. Discussion

The experimental results have shown that the dye patterns, behind a three-dimensional obstacle with a shape of a sphere, an ellipsoid, or a short section of a circular cylinder (the results of which are not shown here) in a linearly stratified fluid, are very similar to the vortex trails over ocean islands as revealed by the satellite pictures. The detailed shape of the body seems to be not important. The only important factor here is the stratification of the fluid If there was no stratification the wake of the three-dimensional obstacle would be three-dimensional in nature and would appear to be turbulent and irregular. Therefore a necessary condition for the appearance of the regular vortex pattern behind an ocean island is either that the atmosphere is stratified, or that an inversion exists The validity of this statement has to await the confirmation by investigating more weather data associated with the actual appearances of the vortex trails. The sufficient conditions for the appearance of these vortex trails appear to be those: there exist a steady wind and one or several inversion layers up to the elevation of the mountain's peak of the island and some cloud forming activities nearby to make the vortex patterns visible.

The phenomenon of vortex trails over ocean islands is associated with a very large Reynolds number which is usually in the range of $10^9 \sim 10^{10}$ based on the horizontal dimensions of the islands. Such a high Reynolds number can never be obtained in the laboratory. For the laboratory experiments shown in Fig.4, 5 and 6, the Reynolds numbers range from 900 to 4,300 which are far below the field values associated with the vortex trails. Then, why there is such a striking resemblance between the laboratory dye pattern and the cloud formation of the vortex trail over the island. Is this just a coincidence? For flows around long cylinders, it is known (Roshko 1954, Gerrard 1966) that periodic vortex shedding from the cylinders exists for Reynolds numbers up to 3×10^5 which is called the critical Reynolds number. No prevailing shedding frequency can be detected for the range $3\times10^5<Re<5\times10^6$. Recently the work of Jones, Cincotta and Walker (1969) gives important clues on the reappearing of regular vortex shedding at a Reynolds number of about $Re=6\sim10\times10^6$. This gives a good indication that periodic vortex shedding probably exists at very high Reynolds numbers. A very similar situation also occurs for the flow past a sphere. Periodic vortex shedding from a sphere exists for Reynolds numbers up to 3×10^5. For $3\times10^5<Re<5\times10^6$ the prevailing shedding frequency can no longer be detected (Achenbach 1974). The question then arises whether periodic vortex shedding from a sphere still exists for Reynolds numbers higher than 5×10^6. Intuitively, we may conjecture that the reappearing of regular vortex shedding at higher Reynolds numbers is highly probable in view of the evidence in the case of the flow around cylinders. Another independent clue on this can also be inferred from the experimental results as follows. Obvervations during the experiments of towing a sphere through a stratified fluid revealed that the streams delineating the stream lines around the nose of a sphere was symmetrical and therefore definitely three-dimensional; this is shown in Fig. 10. In other words for a moderate stratification and a Reynolds number of order 10^4 the vortex is shed from a sphere three-dimensionally. Thereforce, for the vortex trail over the ocean island, the vortex shedding from a three-dimensional island barrier is believed to be three-dimensional for a moderately stratified atmosphere. The startification however quickly and effectively inhibits the vertical motion and the initially three-dimensional turbulent wake collapses and reveals the vertically oriented portion of the vortex structure, reminiscent of the von Kármán vortex street. Thus,the existence of the regular vortex trail coupled with a probable three-dimensional vortex shedding from the island barrier gives a strong support to the hypothesis that periodic vortex shedding from a sphere or other three-dimensional bodies reappears at very high Reynolds numbers. This, of course, still awaits careful experimental verifications.

An attempt will now be made to estimate the vortex spacing of the cloud formation over the island of Madeira, based on the realization that the regular vortex trail as revealed by the satellite picture is a consequence of the stratification of the potential density of the atmosphere. Without stratification the wake was originally three-dimensional and appeared irregular due to turbulence. Thus, to calculate the vortex spacing over the island, experimental results for the vortex shedding from a sphere instead of a long cylinder should be used. If the period of vortex shedding is T and the average velocity in the wake is β U, the vortex would have moved a distance βUT in time T. Hence the spacing of one complete vortex loop is $\lambda = \beta UT$. The frequency f is, then, $f=1/T=\beta U/\lambda$, and the Strouhal number is given by $S=fD/U=\beta D/\lambda$. It follows then $\lambda=\beta D/S$. The wake velocity βU is estimated to be about 0.8 of the free-stream velocity U in the region where the first few vortices appear; then, $\beta=0.8$. The Strouhal number S approaches a value of 0.20 as the Reynolds number reaches 3×10^5 (Achenbach 1974). Since the reappearing of regular vortex shedding from a three-dimensional body for Re$\sim10^{10}$ is highly probable as was indicated in the previous paragraph. As a result, the Strouhal number must also exist for such a high Reynolds number. If we now assume that for Re$\sim10^{10}$ the Strouhal number also equals to 0.20, a value for $Re=3\times10^5$, the vortex spacing is then calculated to be $\lambda=0.8\times40/0.2=160$ km, in which the average width of the island of about 40 km normal to the wind direction is used for D. The actual spacing for the first complete vortex loop measured from Fig. 1. is about 160 km. This excellent agreement between the estimated value and the actual spacing is mainly the consequence of the assumption that the Strouhal number for the case of the vortex trail over the island is 0.20. The exact Strouhal numbers for the flows over islands are yet to be verified by more careful and extensive analysis of field data.

The question then arises why Chopra and Hubert have also obtained good agreement for the spacing ratio between the two-dimensional vortex streets and those over Madeira. The main reason for this is that the Strouhal number for flows around long cylinders remains essentially constant at $S=0.212$ for $1{,}400<Re<3\times10^5$. Such a value was also used by Chopra and Hubert for their calculations. Their value of 0.212 is, of course, very close to the value 0.20 used in our own calculation above. Therefore, we want to support the argument of Moll's that the similarity of these vortex trails to von Kármán vortex streets may not go beyond the general streak-line pattern. In other words, the vortex trails over islands are the vortex loops behind three-dimensional obstacles under the influence of stratification. Thus, they are not von Kármán vortex streets in essence, although they are in appearance

5. Concluding Remarks

For laboratory modeling, many mesoscale atmospheric flow phenomena can be reasonably simulated in the laboratory if done properly. However, caution must be exercised before any definitive conclusions can be made. The key factor here for the laboratory simulation of vortex trails over ocean islands is the stratification of the fluid. It is to be emphasized that laboratory modeling has its limitations, but the first-order solution can often be found if key factors are being identified and incorporated into the model.

In this study, it is demonstrated that the density stratification is indeed the main reason for the appearance of cloud vortex trails over ocean islands. These vortex trails are actually the three-dimensional vortex loops behind the three-dimensional obstacles under the influence of density stratification. Thus, they are not von Kánmán vortex streets in essence, although they are in appearance.

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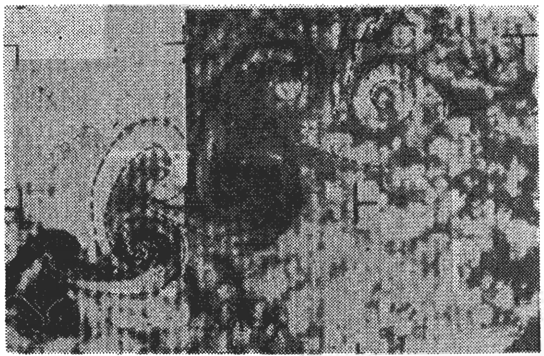


Figure 1. Satellite photograph showing a chain of vortices behind the island of Madeira. The location of the island of Madeira is shown at the lower left corner of the photograph.

Figure 2. A chain of vortices from the lee side of the Canary Islands. The Canary Islands are located outside the upper right corner of the photograph.

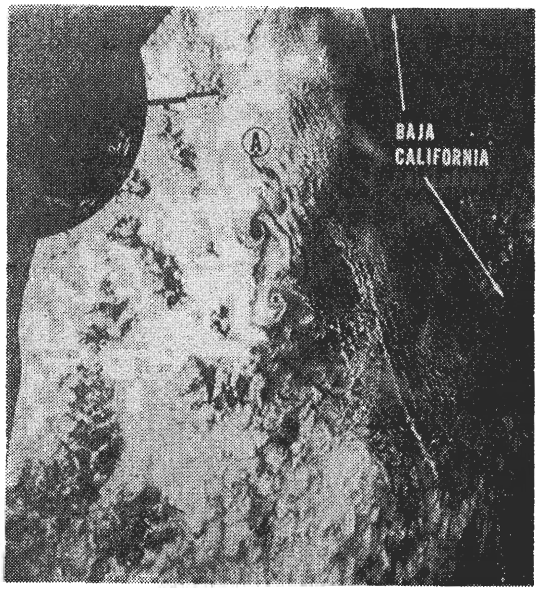
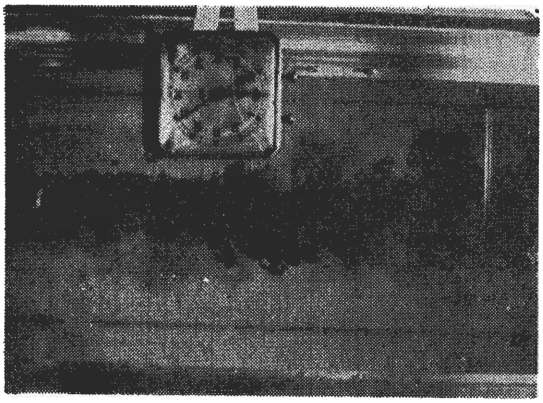


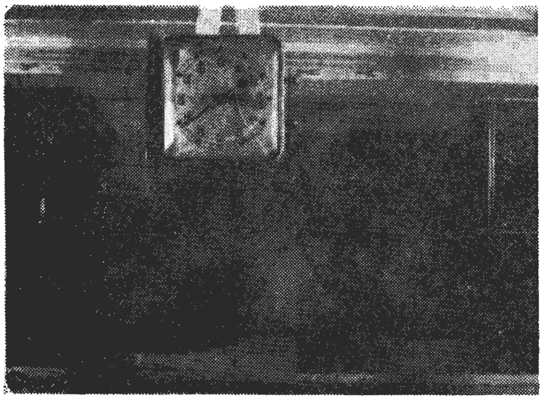
Figure 3. A cloud vortex trail in the downstream of Guadalupe (Baja California). The island of Guadalupe is located at Ⓐ as shown.



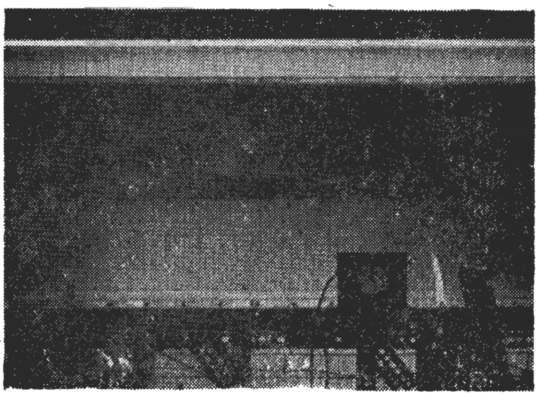
Top view

Side view

(a) Time=25 sec.



Top view



Side view

(b) Time=144 sec.

Figure 4. Formation of turbulent wake and its transformation to an organized quasi-two-dimensional vortex structure during wake collapse. (a) Time=25 sec. (b) Time=144 sec. Experimental conditions: Towing speed $U=2.75$ in/sec; sphere diameter $D=2.25$ in; Brunt-Välsälä frequency $N=0.567$ sec^{-1}; Reynolds No. $Re=UD/\nu=4{,}300$; Richardson No. $Ri=N^2D^2/U^2=0.215$.

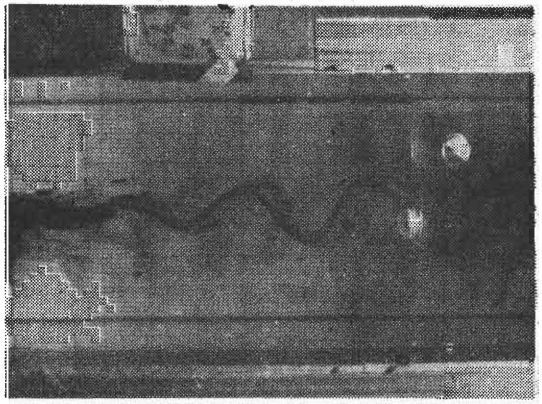


Figure 5. Top view of the wake behind an ellipsoid in a flowing stratified fluid. The ellipsoid is just beyond the left edge of the picture. The flow direction is from the left to the right. Experimental conditions: Flowing speed $U=1.0$ in/sec; ellipsoid diameter $D=2$ in; ellipsoid length $L=8$ in; $N=0.41$ sec^{-1}; $Re=1{,}400$; $Ri=1.64$.

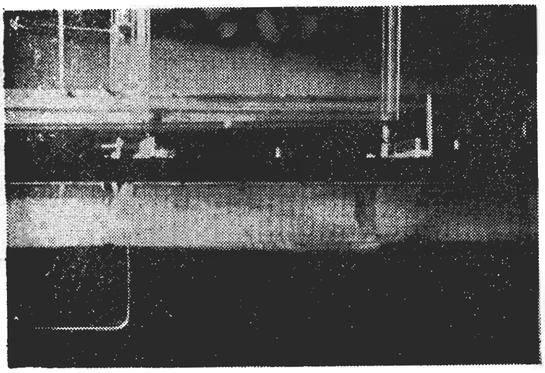


Figure 6. Top and side views of the wake behind a sphere. Experimental conditons: U=0.62 in/sec; D=2.25 in; N=0.41 sec^{-1}; Re=970; Ri=2.1.

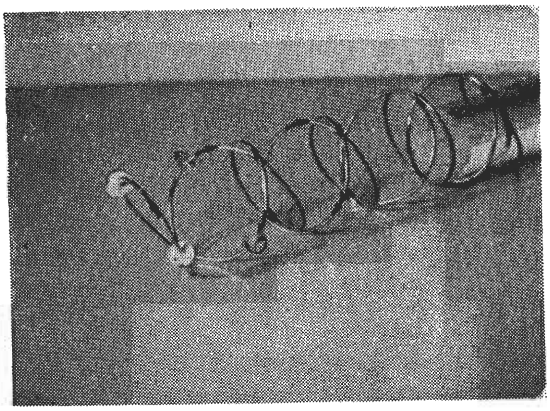


Figure 7. A picture of the close-ended double helix model for the vortex structure in the wake of a sphere. The circular arrows around the vortex tube indicate the circulations while the arrows along the tube represent the vorticity vectors.

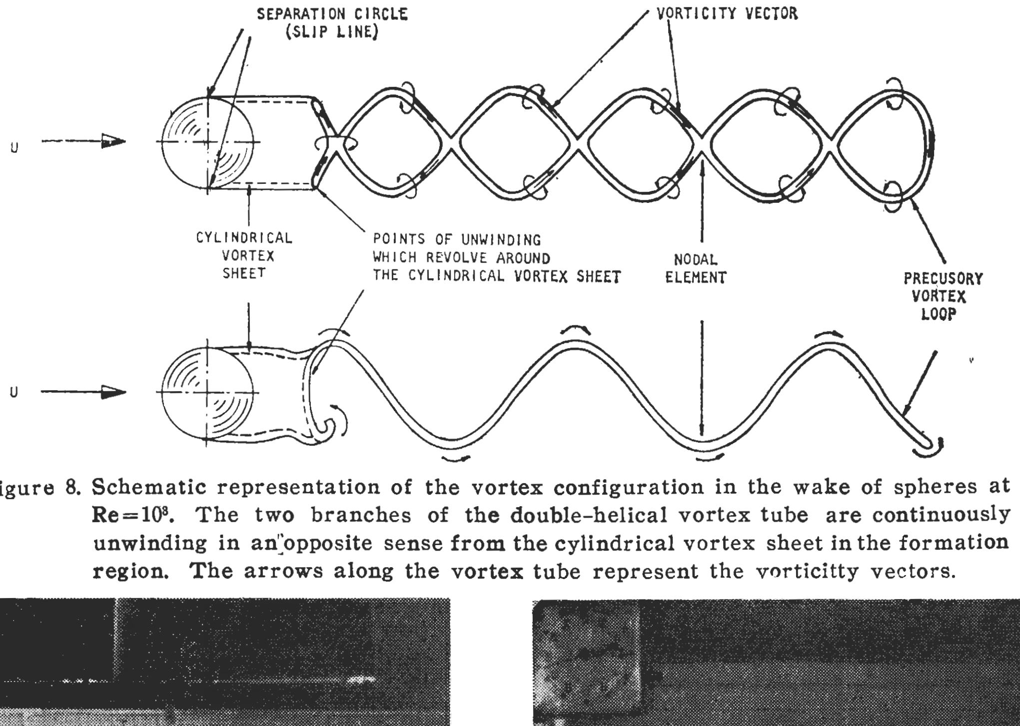
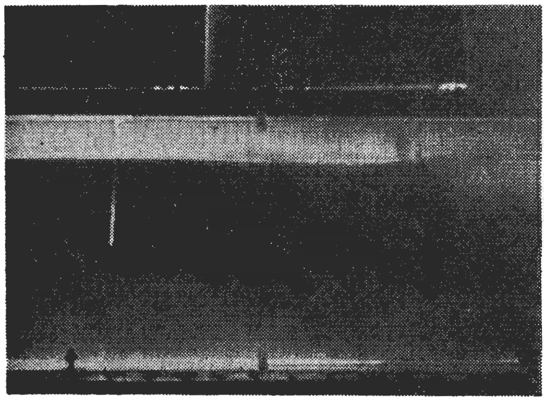
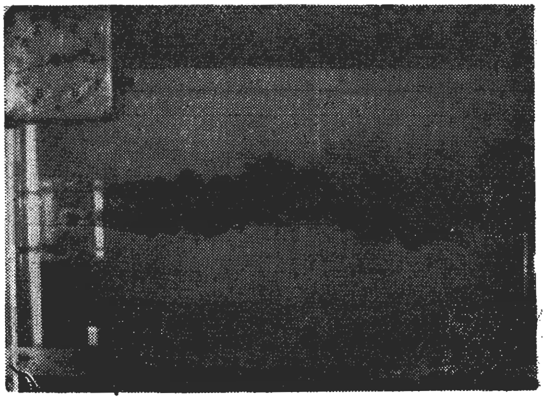


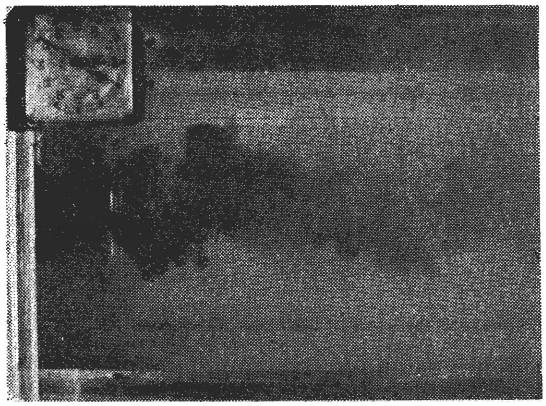
Figure 8. Schematic representation of the vortex configuration in the wake of spheres at Re=10^3. The two branches of the double-helical vortex tube are continuously unwinding in an"opposite sense from the cylindrical vortex sheet in the formation region. The arrows along the vortex tube represent the vorticitty vectors.



(a) Side view
Time=27$\frac{1}{2}$ sec.



(b) Top view
Time=27$\frac{1}{2}$ sec.



(c) Top view
Time=36 sec,

Figure 9. Flow visualization towing experiment in a homogeneous fluid. The dye structure showing the helical nature of the wake. (a) side view; Time=27 sec. (b) Top view; Time=27 sec. (c) Top view; Time=36 sec. Experimental condition $U=2.75$ in/sec; $D=2.25$ in; $Re=4,300$.

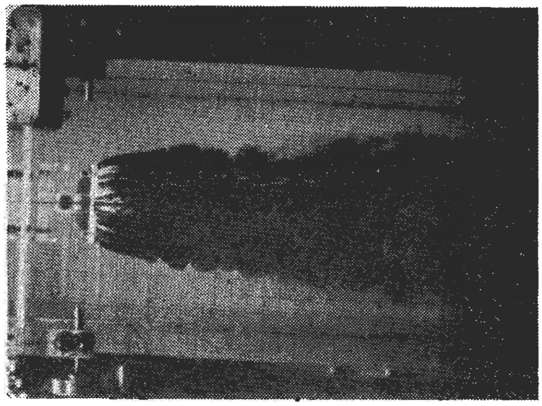


Figure 10. A top view of the dye pattern in the wake of sphere. The vortex shedding is clearly seen to be three-dimensional. Experimental conditon: $U=5.0$ in/sec; $D=5.0$ in; $N=0.802$ sec^{-1}; $Re=17.4\times10^3$; $Ri=0.643$. The picture was taken 22 sec after the commencement of the towing.

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摘要

在本研究中，若于實驗結果經行作出，說明流體密度分層（stratification）爲海洋島嶼後方渦旋列之出現之主要原因。有甚多證據顯示，由於其一般流紋線（streakline）型式近似致使這些渦旋列被認爲范卡門（Von Kármán）渦旋街。實驗顯示，當一球體以定速在層流中拖移時存在有一三維狀之渦旋列逸出（vortex shedding）結構。當流體垂直密度變化不大及欒氏數（Reynolds number）在 10^3 至 10^4 範圍中時。渦旋係作三維狀之逸出。但流體中密度分層，迅速並有效地遏止其垂直向之運動，原始之亂流性尾流（wake）崩潰，並顯示出渦旋結構豎向部份。當自上往下觀察時，似若圓柱後方二維（或水平）之渦旋列型式。有關此三維狀物體層流中渦旋列逸出及其結構，本文並作有頗詳細之論述。

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